PUZZLE SUITE · GAME THEORY NOTES

The Game Theory Behind Word Solver

Information theory for word guessing, and prefix-tree search for word finding

Chafic Charafeddine · chafic.me

A short game-theory primer

Game theory is the mathematics of decisions whose outcome depends on choices, chance, or other people. The ideas below come up throughout this page.

Perfect vs imperfect information

In a game of **perfect information**, every player sees everything that has happened so far, as in chess or Connect 4. In a game of **imperfect information**, something is hidden, such as an opponent's cards in poker. [1]

Zero-sum

A game is **zero-sum** when one player's gain is exactly the other's loss. Most two-player board games and heads-up poker are zero-sum. [2]

Minimax

Minimax means choosing the move whose *worst-case* result is best, on the assumption that the opponent always replies as well as possible. Von Neumann's 1928 minimax theorem showed that every finite two-player zero-sum game has a well-defined value when players are allowed to randomise. [3][4]

Nash equilibrium

A **Nash equilibrium** is a set of strategies in which no player can do better by changing only their own strategy. John Nash proved in 1950 that every finite game has at least one, possibly in mixed (randomised) strategies. In poker this is what "GTO" (game-theory optimal) refers to. [5][6]

Expected value (EV)

The **expected value** is the probability-weighted average outcome of a decision: what it would earn per play over many repetitions. When there is no opponent to outwit, the best play is usually just the play with the highest EV. [7]

Solved games

Following Allis, a game is **ultra-weakly solved** when its result under perfect play is known, **weakly solved** when a strategy that achieves that result from the start is known, and **strongly solved** when perfect play is known from every legal position. [8][9]

Guess helper: five-letter word-guessing games

1 PLAYER · INFORMATION THEORY

1 The game, in plain words

In a **five-letter word-guessing game**, there is a hidden five-letter word, and you have a limited number of guesses (usually six) to find it. Every guess must be a real word. After each guess, every letter is marked in one of

three ways: in the word and in the right spot, in the word but in a different spot, or not in the word at all. You use these clues to narrow down the possibilities until you find the answer.

2 The game theory behind it

The answer is fixed before you start and doesn't react to your guesses, so this is a **single-player search under uncertainty**, not a contest. The theory that fits best is **information theory**, founded by Claude Shannon in 1948. [10]

Every guess sorts the remaining candidate words into groups, one for each colour pattern the guess could produce. With three possible marks for each of five letters, there are at most 3^5 = **243** patterns. A guess is informative when it spreads the candidates evenly over many patterns. **Shannon entropy** measures this exactly:

```
H = − Σ p(pattern) × log₂ p(pattern)
```

H is measured in **bits**. One bit of information halves the number of possibilities on average, so a guess with an expected entropy of 5 bits cuts the candidates by a factor of about 2^5 = 32. [11] This is the approach Grant Sanderson set out in his 3Blue1Brown video lesson: choose the guess with the highest expected information, and weight candidates by how common each word is, since the hidden word is far more likely to be an everyday word than an obscure one. [12]

> **Where game theory comes in:** if the hidden word were chosen by an *adversary* who could change it to dodge your guesses, the right objective would switch from average information to the **worst case**, which is a minimax criterion. That is the reasoning behind Donald Knuth's 1976 algorithm for the code-breaking game Mastermind, which always finds the code in five guesses or fewer. [13]

3 Finding the best play, and how the app does it

- Start with a list of valid words and a prior probability for each one, from word-frequency data.
- For each possible guess, work out which colour pattern it would produce against every remaining candidate, and compute the expected entropy.
- Recommend the guess with the highest score. After the real feedback arrives, keep only the words that match it, and repeat.

IN THE APP

The guess helper ranks guesses by **expected information (Shannon entropy)** over the words that are still possible. Its vocabulary is the free **ENABLE** word list, and each word is weighted by its everyday frequency from the open-source **wordfreq** data, following the 3Blue1Brown approach. [14][15][12]

Word finder: tile word games

COMBINATORIAL SEARCH

1 The game, in plain words

In **tile word games**, each player draws a hand ("rack") of lettered tiles and plays words made from those letters, often on a board. Finding words means solving an anagram problem. Given a set of letters, sometimes including

blank tiles that can stand for any letter, list every valid word that can be spelled with them.

2 The game theory behind it

Full tile word games are multi-player games with **imperfect information** (you can't see your opponents' tiles) and **chance** (which tiles you draw). Strategy involves scoring, board position, and which letters to keep for later. The word finder handles the core **combinatorial search** underneath all of that: listing every legal word your letters allow, which any strategy has to start from.

3 Finding the best play, and how the app does it

Trying every arrangement of the letters is wasteful, because seven tiles have thousands of orderings, most of them not words. A **trie** (prefix tree; Fredkin, 1960) stores the dictionary as a tree in which each path from the root spells the start of a word.[16][17] The search walks down the trie using only the letters still available. It can abandon a branch the moment no word starts with that prefix, so it only explores paths that lead to real words. Blanks are handled by trying each letter at that step.

IN THE APP

The word finder searches the **ENABLE** word list with a **trie**, listing every word that can be made from your letters.[14]

At a glance

Tool	Underlying problem	Opponent?	Key idea	App approach
Guess helper (five-letter word-guessing games)	Single-player search for a hidden word	No (fixed hidden word)	Shannon entropy: expected information per guess	Entropy-ranked guesses; ENABLE list; wordfreq weights
Word finder (tile word games)	Anagram / combinatorial search	The full game has opponents; the search does not	Prefix pruning	Trie search over ENABLE

DETAILS-PAGE BLURB

Word Solver applies information theory to five-letter word-guessing games. It ranks each guess by its expected information in bits, Claude Shannon's entropy, weighted by how common each word is, as in 3Blue1Brown's well-known approach. Its word finder lists every word your tiles can make, using a trie (prefix tree) to search the free ENABLE word list.

References

1. Wikipedia. "Perfect information." https://en.wikipedia.org/wiki/Perfect_information
2. Wikipedia. "Zero-sum game." https://en.wikipedia.org/wiki/Zero-sum_game

3. von Neumann, J. (1928). "Zur Theorie der Gesellschaftsspiele." *Mathematische Annalen* 100, 295–320. https://doi.org/10.1007/BF01448847
4. Wikipedia. "Minimax." https://en.wikipedia.org/wiki/Minimax
5. Nash, J. F. (1950). "Equilibrium points in n-person games." *PNAS* 36(1), 48–49. https://doi.org/10.1073/pnas.36.1.48
6. Wikipedia. "Nash equilibrium." https://en.wikipedia.org/wiki/Nash_equilibrium
7. Wikipedia. "Expected value." https://en.wikipedia.org/wiki/Expected_value
8. Allis, L. V. (1994). *Searching for Solutions in Games and Artificial Intelligence*. PhD thesis, University of Limburg (Maastricht). https://project.dke.maastrichtuniversity.nl/games/files/phd/SearchingForSolutions.pdf
9. Wikipedia. "Solved game." https://en.wikipedia.org/wiki/Solved_game
10. Shannon, C. E. (1948). "A mathematical theory of communication." *Bell System Technical Journal* 27(3), 379–423. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x
11. Wikipedia. "Entropy (information theory)." https://en.wikipedia.org/wiki/Entropy_(information_theory)
12. 3Blue1Brown (G. Sanderson) (2022). Video lesson applying information theory to a five-letter word-guessing game. YouTube. https://www.youtube.com/watch?v=v68zYyaEmEA
13. Wikipedia. "Mastermind (board game)" (Knuth's 1976 five-guess worst-case algorithm). https://en.wikipedia.org/wiki/Mastermind_(board_game)
14. Everything2. "ENABLE word list" (public-domain word list compiled by Alan Beale). https://everything2.com/title/ENABLE+word+list
15. Speer, R. *wordfreq*: word-frequency data for many languages. GitHub. https://github.com/rspeer/wordfreq
16. Fredkin, E. (1960). "Trie memory." *Communications of the ACM* 3(9), 490–499. https://doi.org/10.1145/367390.367400
17. Wikipedia. "Trie." https://en.wikipedia.org/wiki/Trie